So in ſpherical trigonometry, where ſome of the caſes are worked wholly on the ſines, others partly on ſines, and partly on tangents; the extent taken with the compaſſes, between the firſt and ſecond terms, when thoſe terms are of the ſame kind, will reach from the third term to the fourth.

Or the extent from the firſt term to the third, when they are of the ſame kind, will reach from the ſecond term to the fourth.

SECT. XVII.

Some Uſes of the double Scales of Sines, Tangents, and Secants.

PROBLEM XX.

Given the radius of a circle (ſuppoſe equal to 2 *inches) required the ſine, and tangent of* 28° 30′ *to that radius.*

SOLUTION. Open the ſector ſo that the tranſverſe diſtance of 90 and 90, on the ſines; or of 45 and 45 on the tangents; may be equal to the given radius; *viz.* 2 inches: Then will the tranſverſe diſtance of 28° 30′, taken from the ſines, be the length of that ſine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the ſecant of 28° 30′ *was required?*

Make the given radius two inches, a tranſverſe diſtance to 0 and 0, at the beginning of the line of ſecants; and then take the tranſverſe diſtance of the degrees wanted, *viz.* 28° 30′.

A Tangent greater than 45 *degrees (ſuppoſe* 60 *degrees) is found thus.*

Make the given radius, ſuppoſe 2 inches, a tranſverſe diſtance to 45 and 45 at the beginning of the ſcale of upper tangents; and then the required degrees 60° 00′ may be taken from this ſcale.

The ſcales of the upper tangents and ſecants do not run quite to 76 degrees; and as the tangent and ſecant may ſometimes

ſometimes be wanted to a greater number of degrees than can be introduced on the ſector, they may be readily found by the help of the annexed table of the natural tangents and ſecants of the degrees above 75; the radius of the circle being unity.

Degrees.	Nat. Tangent.	Nat. Secant.
76	4,011	4,133
77	4,331	4,445
78	4,701	4,810
79	5,144	5,241
80	5,671	5,759
81	6,314	6,392
82	7,115	7,185
83	8,144	8,205
84	9,514	9,567
85	11,430	11,474
86	14,301	14,335
87	19,081	19,107
88	28,636	28,654
89	57,290	57,300

Meaſure the radius of the circle uſed, upon any ſcale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or ſecant ſought, to be taken from the ſame ſcale of equal parts.

EXAM. *Required the length of the tangent and ſecant of* 80 *degrees to a circle whoſe radius, meaſured on a ſcale of* 25 *parts to an inch, is* 47½ *of thoſe parts.*

Againſt

	tangent.	secant.
Against 80 degrees stands	5,671	5,759
The radius is	47,5	47,5
	28355	28795
	39697	40313
	22684	23036
	269,3725	273,5525

So the length of the tangent on the twenty-fifth scale will be 269 $\frac{1}{3}$ nearly. And that of the secant about 273 $\frac{1}{2}$.

Or thus. The tangent of any number of degrees may be taken from the sector at once; if the radius of the circle can be made a transverse distance to the complement of those degrees on the lower tangent.

EXAM. *To find the tangent of* 78 *degrees to a radius of* 2 *inches.*

Make two inches a transverse distance to 12 degrees on the lower tangents; then the transverse distance of 45 degrees will be the tangent of 78 degrees.

In like manner the secant of any number of degrees can be taken from the sines, if the radius of the circle can be made a transverse distance to the cosine of those degrees. Thus making two inches a transverse distance to the sine of 12 degrees; then the transverse distance of 90 and 90 will be the secant of 78 degrees.

From hence it will be easy to find the degrees answering to a given line, expressing the length of a tangent or secant, which is too long to be measured on those scales, when the sector is set to a given radius.

Thus. For a tangent, make the given line a transverse distance to 45 and 45 on the lower tangents; then take the given radius and apply it to the lower tangents; and the degrees where it becomes a transverse distance is the cotangent of the degrees answering to the given line.

And for a secant. Make the given line a transverse distance to 90 and 90 on the sines. Then the degrees answering to the given radius, applied as a transverse distance on the sines, will be be the co-ſine of the degrees anſwering to the given ſecant line.

PROBLEM XXI.

Given the length of the ſine, tangent, or ſecant, of any degrees; to find the length of the radius to that ſine, tangent, or ſecant.

Make the given length, a tranſverſe diſtance to its given degrees on its reſpective ſcale: Then,

In the ſines. The tranſverſe diſtance of 90 and 90 will be the radius ſought.

In the lower tangents. The tranſverſe diſtance of 45 and 45, near the end of the ſector, will be the radius ſought.

In the upper tangents. The tranſverſe diſtance of 45 and 45 taken toward the center of the ſector on the line of upper tangents, will be the radius ſought.

In the ſecant. The tranſverſe diſtance of 0 and 0, or the beginning of the ſecants, near the center of the ſector, will be the radius ſought.

PROBLEM XXII.

Given the radius and any line repreſenting a ſine, tangent, or ſecant; to find the degrees correſponding to that line.

SOLUTION. Set the ſector to the given radius, according as a ſine, or tangent, or ſecant is concerned.

Take the given line between the compaſſes; apply the two feet tranſverſely to the ſcale concerned, and ſlide the feet along till they both reſt on like diviſions on both legs; then will thoſe diviſions ſhew the degrees and parts correſponding to the given line.

PROBLEM XXIII.

To find the length of a verſed ſine to a given number of degrees, and a given radius.

Make the tranſverſe diſtance of 90 and 90 on the ſines, equal to to the given radius.

Take the tranſverſe diſtance of the ſine complement of the given degrees.

If the given degrees are leſs than 90, the difference between the ſine complement and the radius, gives the verſed ſine.

If the given degrees are more than 90, the ſum of the ſine complement and the radius, gives the verſed ſine.

PROBLEM XXIV.

To open the legs of the ſector, ſo that the correſponding double ſcales of lines, chords, ſines, tangents, may make, each, a right angle.

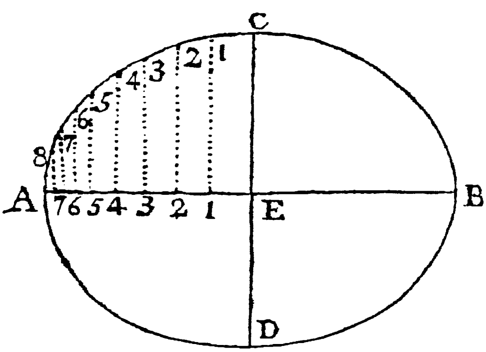
On the lines, make the lateral diſtance 10, a diſtance between 8 on one leg, and ſix on the other leg.

On the ſines, make the lateral diſtance 90, a tranſverſe diſtance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the ſine of any degrees, to their complement.

Or on the ſines, make the lateral diſtance of 45 a tranſverſe diſtance between 30 and 30.

PROBLEM XXV.

To deſcribe an Ellipſis, having given AB *equal to the longeſt diameter; and* CD *equal to the ſhorteſt diameter.*



SOLUTION. 1ſt. Set the two diameters AB, CD, at right angles to each other in their middles at E.

2d. Make AE a tranſverſe diameter to 90 and 90 on the ſines; and take the tranſverſe diſtances of 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, ſucceſſively, and apply thoſe diſtance to AE from E towards A, as at the points 1, 2, 3, 4, 5, 6, 7, 8; and thro' thoſe

thoſe points draw lines parallel to EC.

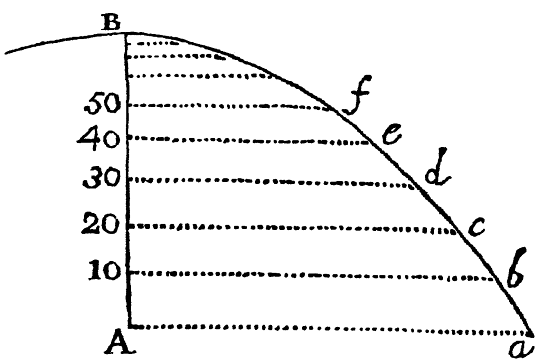
3d. Make EC a tranſverſe diſtance to 90 and 90 on the ſines; take the tranſverſe diſtances of 80°, 70°, 60°, 50°, 40°, 30°, 20°, 10°, ſucceſſively, and apply thoſe diſtances to the parallel lines from 1 to 1, 2 to 2, 3 to 3, 4 to 4, 5 to 5, 6 to 6, 7 to 7, 8 to 8, and ſo many points will be obtained thro' which the curve of the ellipſis is to paſs.

The ſame work being done in all the four quadrants, the elliptical curve may be completed.

This Problem is of conſiderable uſe in the conſtruction of ſolar Eclipſes; but inſtead of uſing the ſines to every ten degrees, the ſines belonging to the degrees and minutes correſponding to the hours, and quarter hours are to be uſed.

PROBLEM XXVI.

To deſcribe a Parabola whoſe parameter ſhall be equal to a given line.



SOLUTION. 1ſt. Draw a line to repreſent the axis, in which make AB equal to half the given parameter; divide AB like a line of ſines to every ten degrees, as to the points 10, 20, 30, 40, 50, *&c.* and thro' theſe points draw lines at right angles to the axis AB.

2d. Make the lines A*a*, 10*b*, 20*c*, 30*d*, 40*e*, *&c.* reſpectively equal to the chords of 90°, 80°, 70°, 60°, 50°, *&c.* to the radius AB, and the points *a, b, c, d, e, &c.* will be in the curve of the parabola.

The like work may be done on both ſides of the axis when the whole curve is wanted.

As the chords on the ſector run no farther than 60°, thoſe of

70°,